Пример:

Задача 2:

$$J[y(x)] = \int_0^1 \sqrt{y(1+y'^2)}\,dx$$

$$y(0) = y(1) = \frac{1}{\sqrt{2}}$$

Сост. ур-е Эйлера:

$$-\frac{d}{dx}\left(\frac{1}{\cancel{2}\sqrt{y(1+y'^2)}}\cdot\cancel{2}y'y\right) + \frac{1+y'^2}{2\sqrt{y(1+y'^2)}} = 0$$

Производная:

$$\frac{-(y''y + y'y')\sqrt{y(1+y'^2)} - \frac{yy'}{2\sqrt{y(1+y'^2)}}(y'^3 + 2yy'y'')}{(y(1+y'^2))} + \frac{1+y'^2}{2\sqrt{y(1+y'^2)}} = 0$$

$$+\frac{-2\left(y''y+y'^2\right)\cdot y\left(1+y'^2\right)-yy'^2-yy'^2-2y^2y'^2y''}{2\left(y\left(1+y'^2\right)\right)^{3/2}}+$$

$$-\frac{1+y'^2\sqrt{y(1+y'^2)}}{2\sqrt{y(1+y'^2)}}=0$$

$$2\left(y''y^2+\cancel{y''y^2y'^2}+\cancel{yy'^2}+\cancel{yy'^4}\right)-yy'^2-\cancel{yy'^4}-\cancel{2y^2y'^2y''}-$$

$$-y-\underbrace{\cancel{yy'^2}-yy'^2}_{-2yy'^2}-yy'^4$$

$$2y''y^2-yy'^2-y=0$$

$$y(2y''y-y'^2-1)=0$$

$$y=0$$

~~[illegible]~~

$$y'=p(y)\qquad y''=\frac{dp}{dy}\cdot\frac{dy}{dx}$$

$$\cancel{2\frac{dp}{dy}\cdot\frac{dy}{dx}}\qquad y''=p\frac{dp}{dy}$$

$$2p\frac{dp}{dy}\cdot y-p^2-1=0$$

$$2p\frac{dp}{dy}\cdot y=p^2+1$$

$$\frac{2p\,dp}{p^2+1}=\frac{dy}{y};\quad \ln(p^2+1)=\ln y+\ln x$$

$$p^2+1=c\cdot y$$

$$\Rightarrow p=\sqrt{cy-1}$$

$$y'=\sqrt{cy-1}$$

$$\frac{dy}{\sqrt{cy-1}}=dx$$

$$(cy-1)^{-\frac{1}{2}}dy=x+C_1$$

$$\frac{2}{c}\sqrt{cy-1}=x+C_1$$

$$\frac{4}{c^2}(cy-1)=(x+C_1)^2$$

$$\frac{4}{c}y=(x+C_1)^2+\frac{4}{c^2}$$

$$y=\frac{c}{4}(x+C_1)^2+\frac{1}{c}$$

$$y(0)=\frac{c}{4}C_1^2+\frac{1}{c}=\frac{1}{\sqrt{2}}$$

$$y(1)=\frac{c}{4}(1+C_1)^2+\frac{1}{c}=\frac{1}{\sqrt{2}}$$